Application of degenerated hexahedral Whitney elements in the modeling of NDT induction thermography of laminated CFRP composite

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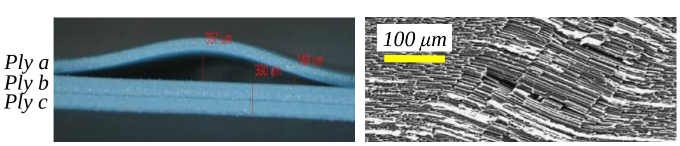
In this paper, degenerated hexahedral Whitney elements are used to modeling Carbon Fiber Reinforced Polymer (CFRP) composites in Non Destructive Testing (NDT) induction thermography technique. These elements avoid meshing the thin regions of composite and take into account the anisotropic multi-layer characteristic of the materials and also the flaws inside their volume. The eddy-current problem is solved using $A-\phi$ formulation. The accuracy of this method is shown by comparison with classical hexahedral elements.

***Index Terms*—Laminated CFRP, NDT, Induction Thermography, Thin Region, Anisotropic material, Degenerated Whitney element.**

I. Introduction

Thank to their excellent performance laminated CFRP composites are widely used in aeronautics industry. These material are a stacking of anisotropic unidirectional plies in which carbon fibers are oriented in a same direction. In order to ensure the mechanical properties, the orientation of each ply in a laminate may be different. The stacking sequence describes the orientation of all plies of the laminate. The thickness of a ply (about $150\mu m$) is very small compared to the size of an entire plate which is generally in order of some meters.

Throughout the life cycle of the material, the NDT methods allow controlling their quality. These methods detect and characterize flaws in the composite, such as delaminations, porosity, fiber rupture (Fig. 1). In order to detect flaws, induction thermography method is based on the effect of the disturbance of eddy-current and heat flow due to the presence of the flaw. As consequence, in order to evaluate precisely the performance of the technique, the distribution of induced currents in the laminate and their interaction with the flaws must be precisely modeled. The distribution of eddy-current depends on the stacking sequence of the material. The disturbance of eddy-current depends on the nature of the flaw.



(a) Delamination between two plies *a* and *b*

(b) Fiber rupture

Fig. 1. Studied flaws.

In [1], delamination and fiber rupture flaws are modeled using classical hexahedral finite element model where each ply volume is meshed. However, meshing thin regions generates generally distorted elements which effect on the convergence and computation time.

In this paper, in order to improve this model, an extension of single layer degenerated prismatic Whitney elements method proposed by Ren [2] to multi-layer anisotropic regions is presented. Double layers shell elements degenerated from hexahedron elements will be firstly presented. This is followed by models of different flaw types. The influence of fiber rupture and delamination flaws on the circulation of eddy-current will be shown. Results obtained with degenerated elements will be validated by comparisons with hexahedron elements.

II. Degenerated Elements

The degenerated nodal elements are determined by:

$$W_{nS}^{-,+} = (W_n^{2D}\beta^-, W_n^{2D}\beta^+) \tag{1}$$

where W_n^{2D} denotes the nodal elements defined on the quadrilateral mesh of the upper and lower surfaces of the thin region which are:

$$\lambda_1 = (1-u)(1-v)/4, \lambda_2 = (1+u)(1-v)/4 \tag{2}$$

$$\lambda_3 = (1+u)(1+v)/4, \lambda_4 = (1-u)(1+v)/4 \tag{3}$$

$\beta^- = (1-w)/2$ et $\beta^+ = (1+w)/2$ are the interpolation functions along the thickness of the region.

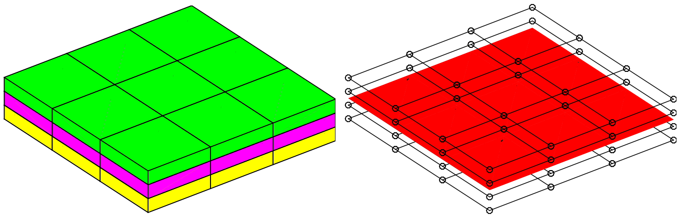
The degenerated edge elements are written as follows:

$$\boldsymbol{W}_{aS}^{-,+,\pm} = (\boldsymbol{W}_a^{2D}\beta^-, \boldsymbol{W}_a^{2D}\beta^+, W_n^{2D}\boldsymbol{n}/\varepsilon) \tag{4}$$

$\boldsymbol{W}_a^{2D}$ is the set of edge elements defined on a quadrilateral.

III. Modeling of anisotropic multi-layer region

In the case of a multi-layer region, it is necessary to create bi-layer quadrilateral meshes that represent each layer of the region (Fig. 2). The integrals calculated on the volume of each layer are transformed into surface integrals calculated on both surfaces of the corresponding bi-layer quadrilateral mesh. The coefficients associated with the common edges and common



(a) Volume mesh. (b) Surface bi-layer meshes.

Fig. 2. Bi-layer meshes of a multi-layer region.

nodes between two bi-layer meshes is the contribution of those calculated in each mesh. The left-hand side of the $\boldsymbol{A}-\phi$ formulation in the laminate domain Ω_c [3] can be determined by:

$$\int_{\Omega_c} \frac{1}{[\mu]} (\boldsymbol{\nabla} \times \boldsymbol{W}_a)(\boldsymbol{\nabla} \times \boldsymbol{A}) d\Omega =$$
$$\sum_{i=1,k} \int_{\Gamma} \frac{1}{\varepsilon_i \mu_{z,i}} [\boldsymbol{n} \times \boldsymbol{W}_a^{2D}][\boldsymbol{n} \times \boldsymbol{A}] d\Gamma +$$
$$\sum_{i=1,k} \int_{\Gamma} \varepsilon_i \frac{1}{[\mu]_{xy,i}} \left(\int_{-1}^{+1} < \boldsymbol{\nabla} \times \boldsymbol{W}_a^{2D} > \right.$$
$$\left. < \boldsymbol{\nabla} \times \boldsymbol{A} > dw \right) d\Gamma$$

$$\int_{\Omega_c} [\sigma] \left(\boldsymbol{W}_a + \boldsymbol{\nabla} W_n\right) \left(\boldsymbol{A} + \boldsymbol{\nabla}\phi\right) d\Omega =$$
$$\sum_{i=1,k} \int_{\Gamma} \frac{\sigma_{z,i}}{\varepsilon_i} \left(\varepsilon_i W_{an} + [W_n^{2D}]\right) \left(\varepsilon_i A_n + [\phi]\right) d\Gamma +$$
$$\sum_{i=1,k} \int_{\Gamma} \varepsilon_i [\sigma]_{xy,i} \left(\int_{-1}^{+1} < \boldsymbol{W}_a^{2D} + \boldsymbol{\nabla} W_n^{2D} > \right.$$
$$\left. < \boldsymbol{A} + \boldsymbol{\nabla}\phi > dw \right) d\Gamma \tag{5}$$

where k is the number of degenerated elements, ε_i the thickness of element i, Γ the median surface of the laminate, $\mu_{z,i}$ and $\sigma_{z,i}$ the permeability and the electrical conductivity in the normal direction to the median surface of the material, $[\mu]_{xy,i}$ and $[\sigma]_{xy,i}$ the physical tensors of the material defined in the plan of its median surface. In (5), one can set $\sigma_z = 0$ when neglecting the normal component of eddy-current as used in [2][4]. $< f >= f_{i \in E^-} \beta^- + f_{i \in E^+} \beta^+$ and $[f] = f_{i \in E^+} - f_{i \in E^-}$ denote respectively the weighted average of f over the thickness of the layer and the jump of f across the layer.

IV. Flaw model

To integrate the flaws in our model, the physical properties are locally modified in the flaws region according to the nature of flaws. For example, in the case of fiber rupture, the carbon fibers are interrupted. The current is forced to pass through plies and deviated when passing the defect. In the case of delamination, the fibers are not unbroken but undulated; the circulation of induced currents is uninterrupted in the direction of fibers. More details will be provided in the final paper.

V. Comparison with hexahedron elements

A wired rectangular-shaped coil is used to inspect a 8 plies composite laminate. The stacking sequence of the laminate is $[135°/90°/45°/0°/0°/45°/90°/135°]$. A fiber rupture are introduced only in the first $0°$ ply (the fourth with respect to the bottom one). The current intensity in the coil is $100A$ at $1MHz$. The thickness of a ply h_{ply} is $136\mu m$. The thickness of inserted flaws h_f is $68\mu m$ or haft of h_{ply}. The dimensions and position of flaws are given in the Fig. 3.

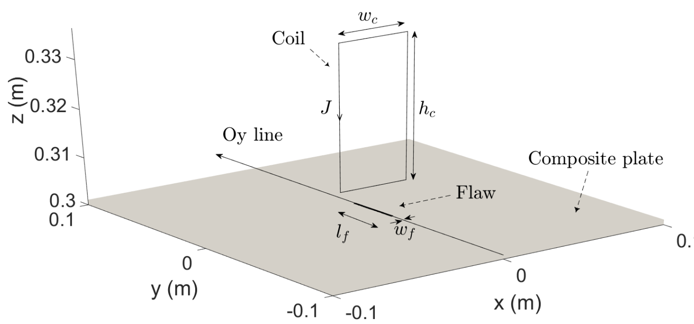


Fig. 3. Induction thermography inspection set-up of a laminated CFRP composite using a rectangular shaped coil.

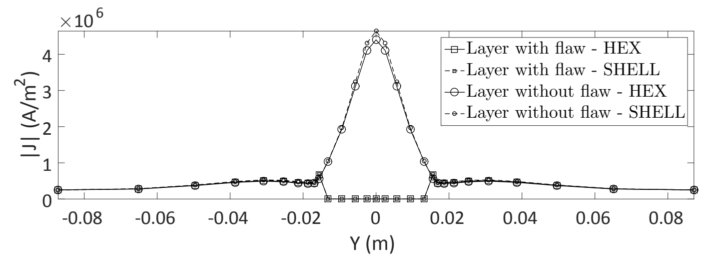


Fig. 4. Variation of average current density following the Oy line in the $0°$ ply containing the flaw.

The Figs. 4 show the variation of induced current density in the $0°$ ply containing the flaw. The behavior of eddy-currents is correctly simulated using shell elements. The results show a good accordance between two methods.

Degenerated hexahedral elements can be used to modeling anisotropic multi-layer composite containing flaws. In the presented cases, the use of these elements can achieve the same accuracy as hexahedron one with reduced computation time. In the final paper, this model will be coupled with a thermal one. Simulation results in the case of delamination will be shown and compared with classical elements. An application of shell elements in the modeling of NDT induction thermography will be presented. Eddy-current and thermal behaviors with the presence of flaws of different natures will be discussed.

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